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RESEARCH ON MEASURING THERMOPHYSICAL PROPERTIES USING PARAMETER ESTIMATION

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In this paper a new prompt method is proposed for the measurement of thermal conductivity k and thermal diffusivity α which are functions of temperature. This method is based on parameter estimation. The k and α can be got by measurement of the temperature on sample and heat-flow through the sample. The experiment has also confirmed that the method is feasible.

Parameter estimation method provides the tools for efficient use of the sampled data to estimate coefficient appearing mathematical models. The principle of the measuring thermophysical properties mentioned in this paper is based on the inverse problem of the heat conduction equation, i.e. the special solution of this equation is used to determine coefficent appearing in the equation. For one dimension heat conduction system, there are

$$\frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) = C_p\rho\frac{\partial T}{\partial t} \qquad (1)$$

$$T = f_1(t) \qquad x = 0,\ t > 0 \qquad (2)$$

$$T = f_2(t) \qquad x = L,\ t > 0 \qquad (3)$$

$$k\frac{\partial T}{\partial x} = f_3(t) \qquad x = L,\ t > 0 \qquad (4)$$

$$T = T_0 \qquad t = 0 \qquad (5)$$

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where x is space variable, T is temperature, t is time. k, C_p, ρ and L are thermal conductivity, heat capacity, density and thickness of sample respectively. T_0 is room temperature and $f_1(t)$, $f_2(t)$ and $f_3(t)$ are known boundary conditions.

According to the theory of non-linear partial differential equation, k and $C_p\rho$ can be got when three boundary conditions and initial condition are known. In pratice, it always needs to establish a target function for estimating k and $C_p\rho$. In the case of Eqs (1-5), a target function is generally expressed as

$$J=\int_0^{\tau}(T\big|_{x=0}-\hat{T}\big|_{x=0})^2\,dt \tag{6}$$

where τ is length of working time in system. $T|_{x=0}$ and $\hat{T}|_{x=0}$ are practical-boundary temperature and estimated boundary temperature respectively. J is the function of k and $C_p\rho$ due to $\hat{T}|x=0$ is the function of k and $C_p\rho$. And J can be decreased step by step during the process of searching for k and $C_p\rho$. k and $C_p\rho$ which make J reach its minimal value are considered as real value of k and $C_p\rho$.

During the process of searching for optimal value of k and $C_p\rho$, it exists a problem of solving non-linear partial differential equation. Numerical solution or approximated solution are often used in engineering, for example, finite element method, finite difference method, Garlekin method, orthogonal collocation method, weighted-residual method and block-pulse operator [1-2], but it will spend a certain time to solve partial differential equation. As a result it needs a lot of time to obtain the optimal value because several tens steps will be used in the searching process. Now if the Kirchoff transform and Taylor expansion are adopted, the boundary value $\hat{T}|_{x=0}$ will be approximately got without solving non-linear partial differential equations. So it will make the time of estimating k and $C_p\rho$ much less than before and has practical value.

In this paper, Kirchoff transform (3) is used in Eq. (1), that is T is replaced by U. The replacement equation is as following

$$U=\int_{T_o}^{T}\frac{k}{k_0}\,dT' \tag{7}$$

where k_0 is k at 0° temperature. Then Eq. (1) become following

$$\frac{\partial^2 U}{\partial x^2} = \frac{1}{\alpha}\frac{\partial U}{\partial t} \tag{8}$$

where α as the function of temperature is thermal diffusivity, that is, $k/(C_p \rho)$.

In a given interval of temperature, k can be written as

$$k = k_0 (1 + b_1 T + b_2 T^2 + b_3 T^3) \tag{9}$$

where k_0, b_1, b_2 and b_3 are all constant. The boundary conditions and initial conditions Eqs (2-5) can be obtained by Eq. (7) and Eq. (9) as

$$U = f_1(t) - T_0 + (b_1/2)(f_1^2(t) - T_0^2) + (b_2/3)(f_1^3(t) - T_0^3) + (b_3/4)(f_1^4(t) - T_0^4)$$

$$x = 0,\ t > 0 \tag{10}$$

$$U = f_2(t) - T_0 + (b_1/2)(f_2^2(t) - T_0^2) + (b_2/3)(f_2^3(t) - T_0^3) + (b_3/4)(f_2^4(t) - T_0^4)$$

$$x = \mathrm{L},\ t > 0 \tag{11}$$

$$k_0 \frac{\partial U}{\partial x} = f_3(t) \qquad x = L,\ t > 0 \tag{12}$$

$$U = 0 \qquad t = 0 \tag{13}$$

Now Taylor expansion is used to U at $x = 0$ and take first five items, then

$$\hat{U}\Big|_{x=0} = U\Big|_{x=L} + L\frac{\partial U}{\partial x}\Big|_{x=L} + \left(L^2/2!\right)\frac{\partial^2 U}{\partial x^2}\Big|_{x=L} +$$

$$+ \left(L^3/3!\right)\frac{\partial^3 U}{\partial x^3}\Big|_{x=L} + \left(L^4/4!\right)\frac{\partial^4 U}{\partial x^4}\Big|_{x=L} \tag{14}$$

Resuming

$$\frac{1}{\alpha} = a_0 + a_1 U + a_2 U^2 + a_3 U^3 \tag{15}$$

in a given interval of temperature, then following equation can be got by Eq. (11) and Eq. (12)

$$U|_{x=L} =$$

$$= f_2(t) - T_0 + (b_1/2)(f_2^2(t) - T_0^2) + (b_2/3)(f_2^3(t) - T_0^3) + (b_3/4)(f_2^4(t) - T_0^4) \tag{16}$$

$$\frac{\partial U}{\partial x}\Big|_{x=L} = \frac{1}{k_0} f_3(t) \tag{17}$$

$$\frac{\partial^2 U}{\partial x^2}\Big|_{x=L} = (a_0 + a_1 U + a_2 U^2 + a_3 U^3)\frac{\partial U}{\partial t}\Big|_{x=L} \tag{18}$$

$$\frac{\partial^3 U}{\partial x^3}\Big|_{x=L} = (a_0 + a_1 U + a_2 U^2 + a_3 U^3)\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial x}\right)\Big|_{x=L} + $$

$$+ (a_1 + 2a_2 U + 3a_3 U^2)\frac{\partial U}{\partial x}\frac{\partial U}{\partial t}\Big|_{x=L} \tag{19}$$

$$\frac{\partial^4 U}{\partial x^4}\Big|_{x=L} = (a_0 + a_1 U + a_2 U^2 + a_3 U^3)^2 \frac{\partial^2 U}{\partial t^2}\Big|_{x=L} + $$

$$+ 2(a_1 + 2a_2 U + 3a_3 U^2)\frac{\partial U}{\partial x}\frac{\partial}{\partial t}\left(\frac{\partial U}{\partial x}\right)\Big|_{x=L} + $$

$$+ 2(a_1 + 2a_2 U + 3a_3 U^2)(a_0 + a_1 U + a_2 U^2 + a_3 U^3)\left(\frac{\partial U}{\partial t}\right)^2\Big|_{x=L} + $$

$$+ (2a_2 + 6a_3 U)\left(\frac{\partial U}{\partial x}\right)^2 \frac{\partial U}{\partial t}\Big|_{x=L} \tag{20}$$

Substituting in Eq. (14) from Eqs (16-20), $\hat{U}|_{x=0}$ can be given. Now taking target function as

$$J = \sum_{i=1}^{N}\left(U_i\Big|_{x=0} - U_i\Big|_{x=0}\right)^2 \tag{21}$$

where N is the sampling number. $k_0, b_1, b_2, b_3, a_0, a_1, a_2$ and a_3 are considered as parameters. Then J is the function of parameters. The real k and α, as the functions of temperature, can be expressed like Eq. (9) and Eq. (15) when J reaches the optimal.

The calculation of heat flow and optimization of target function

In this paper, reference sample is used to obtain the value of boundary heat flow. Temperature field in the internal of reference sample and heat flow of boundary can be calculated by known k, α of the reference sample and $T2$, $T3$ (shown in Fig. 1).

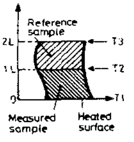

Fig. 1 Measured temperature points on sample surface

If measured sample is joined tightly with the reference sample, the heat flow of boundary on measured sample is equal to one of boundary on reference sample.

The non-linear heat condition equation must be solved when calculating heat flow of boundary on reference sample. Crank-Nicolson finite differential method [4] is chosen to solve the problem in this paper, because of considering accuracy and stability. Then Eq. (8) will become

$$\frac{1}{a_j}\frac{U_j^{n+1} - U_j^n}{\Delta t} = \frac{U_{j-1}^{n+1} - 2U_j^{n+1} + U_{j+1}^{n+1}}{\Delta x} \tag{22}$$

In fact, Crank-Nicolson method is one that uses explicit method and implicit method alternately. Its advantage is not only the stability of all sizes of time step (dt), but also the higher accuracy than explicit method and implicit method used only.

In order to make the target function reach its minimal point as quick as possible, it is proposed to use improved Levenberg-Marquardt-Fletcher method [5] which is one of non-linear least square method. Its convergency speed is faster several tens times than that of simplex method. But sometimes it will also happen that convergency speed of parameter become much slow because of choosing unadapted initial value. According to this case two-steps optimal searching method is used. That is, taking b_1, b_2, b_3, a_1, a_2 and a_3 as zero, first to search k_0 and a_0 only and second to search optimal point for eight variable together after partial optimal point of k_0 and a_0 has searched. This method has been confirmed that it can solve the problem mentioned before.

However, the convergency speed of this method will get slow when target function approaches its optimal point. So the second method which searches for optimal point by inconstant step is proposed in this paper. Meanwhile some rules of using the method are also proposed.

The instrument based on above statemenet to measure k and α has been made using APPLE-II microcomputer. It can realize the sampling $T1$, $T2$ and $T3$ at given times and finish a series of calculation and display the values of k and α on CRT or printer. The block diagram of the instrument is shown by Fig. 2.

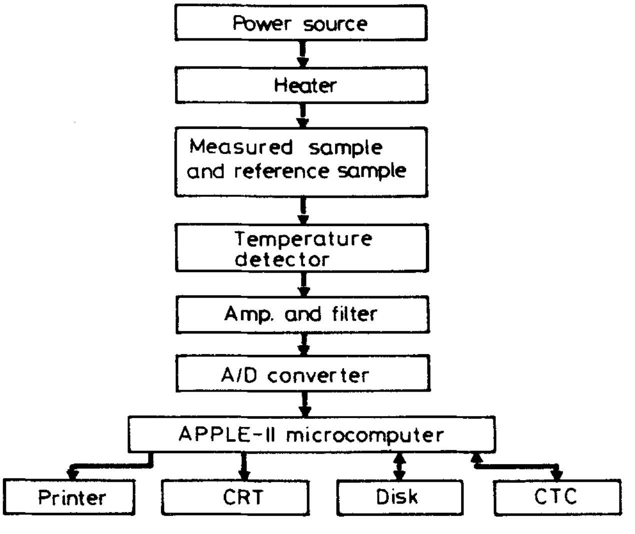

Fig. 2 The block diagram of thermal physical property measurement

In order to improve accuracy, a program of non-linearity correction and temperature lag for thermocouple and the digital filters are made. So the error of measuring system will be decreased.

Results

Two kinds of the samples have been measured. The results are as following (about taking half hour for one experiment). Estimation formula of measuring water glass pearlite by the instrument is expressed as:

$$k = 0.06673 + 4.698 \times 10^{-4} T + 1.352 \times 10^{-6} T^2 + 1.263 \times 10^{-10} T^3 \quad (23)$$

$$\frac{1}{\alpha} = (2.325 - 5.054 \times 10^{-3} U + 1.344 \times 10^{-8} U^2) \times 10^6 \quad (24)$$

There are compared results of k and α in Table 1. $\hat{k}$ and $\hat{\alpha}$ are got by Eq. (22) and Eq. (23). Another one comes from Shanghai Research Institute of Building Science which use the pulse method to measure k point by point and East China University of Chemical Technology which use fall-down method to measure $C_p \rho$ on correspond points and then α is calculated by k and $C_p \rho$.

The value measured rigid polyurethane foamed plastic is shown as (compared on 30 only).

Pulse method combined fall-down method

$$k = 0.02096 \text{ w/mk}, \quad \alpha = 3.206 \times 10^{-7} \text{ m}^2/\text{s}$$

Parameter estimation method

$$k = 0.02090 \text{ w/mk}, \alpha = 3.239 \times 10^{-7} \text{ m}^2/\text{s}$$

Mean square deviation

$$\sigma_k = 2.56 \times 10^{-1} \text{ w/mk}$$

$$\sigma_\alpha = 1.21 \times 10^{-8} \text{ m}^2/\text{s}$$

It has been seen from above that parameter estimation method can be used to measure the value of k and α as function of the temperature in one experiment. It's important that this new method saves the measuring time compared with traditional methods. In another words, the method has advantages of simple structure, convenient operation and cheap price.

Table 1 The comparison of k and α between parameter estimation method and double-plate method for water glass pearlite

T, °C	$k\times10^2$ * w/mk	$\hat{k}\times10^2$, ** w/mk	err, %	$\alpha\times10^7$,* m^2/s	$\hat{\alpha}\times10^7$, ** m^2/s	err, %
15	6.561	6.773	3.23	4.305	4.301	–.0860
20	6.619	6.809	2.87	4.351	4.339	–.0683
25	6.667	6.848	2.54	4.398	4.396	–.0529
30	6.735	6.886	2.22	4.445	4.443	–.0392
35	6.795	6.925	1.92	4.492	4.490	–.0270
40	6.854	6.967	1.65	4.538	4.537	–.0157
45	6.914	7.010	1.39	4.584	4.584	0
50	6.974	7.055	1.15	4.630	4.631	0.0050
55	7.035	7.100	0.928	4.676	5.677	0.0165
60	7.096	7.147	0.725	4.721	4.722	0.0282
65	7.157	7.196	0.538	4.765	4.767	0.0411
70	7.219	7.246	0.369	4.809	4.812	0.0556
75	7.281	7.297	0.217	4.852	4.855	0.0722
80	7.344	7.350	0.080	4.894	5.898	0.0913
85	7.407	7.404	–.041	4.935	4.940	0.134
90	7.470	7.459	–.145	4.974	4.981	0.139
95	7.534	7.516	–.235	5.012	5.021	0.168
100	7.598	7.574	–.310	5.049	5.059	0.201

*double-plate method

**parameter estimation method

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Zusammenfassung — Es wird ein neues Verfahren zur Schnellbestimmung der Wärmeleitfähigkeit k und der Temperaturleitfähigkeit α beschrieben, welche beides eine Funktion der Temperatur darstellen. Diesem Verfahren liegt eine Parameterschätzung zugrunde. k und α können durch Messung der Temperatur an einer Probe bzw. des Wärmeflusses durch eine Probe ermittelt werden. Die Experimente erwiesen die Anwendbarkeit dieses Verfahrens.